Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced

Thursday 24 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The line $x = 8$ is a directrix of the ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > 0, \quad b > 0,$$

and the point (2, 0) is the corresponding focus.

Find the value of a and the value of b.

(5)

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2. Use calculus to find the exact value of $\displaystyle\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\,\mathrm{d}x$.

(5)

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3. (*a*) Starting from the definitions of sinh x and cosh x in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(*b*) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

4.

$$I_n = \int_0^a (a - x)^n \cos x \;\mathrm{d}x, \quad a \geq 0, \quad n \geq 0.$$

(*a*) Show that, for $n \geq 2$,

$$I_n = na^{n-1} - n(n-1)I_{n-2}$$

(5)

(*b*) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \,\mathrm{d}x$.

(3)

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5. Given that $y = (\text{arcosh } 3x)^2$, where $3x > 1$, show that

(*a*) $(9x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 36y,$

(5)

(*b*) $(9x^2 - 1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18.$

(4)

6.

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ is an eigenvector of **M**,

(*a*) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$,

(2)

(*b*) show that $k = 3$,

(2)

(*c*) show that **M** has exactly two eigenvalues.

(4)

A transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by **M**.

The transformation T maps the line l_1, with cartesian equations $\frac{x-2}{1} = \frac{y}{-3} = \frac{z+1}{4}$, onto the line l_2.

(*d*) Taking $k = 3$, find cartesian equations of l_2.

(5)

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7. The plane Π has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

(*a*) Find an equation of Π in the form $\mathbf{r.n} = p$, where $\mathbf{n}$ is a vector perpendicular to Π and p is a constant.

(5)

The point P has coordinates (6, 13, 5). The line l passes through P and is perpendicular to Π. The line l intersects Π at the point N.

(*b*) Show that the coordinates of N are (3, 1, –1).

(4)

The point R lies on Π and has coordinates (1, 0, 2).

(*c*) Find the perpendicular distance from N to the line PR. Give your answer to 3 significant figures.

(5)

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8. The hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

The line l_1 is the tangent to H at the point P ($4 \sec t$, $2 \tan t$).

(*a*) Use calculus to show that an equation of l_1 is

$$2y \sin t = x - 4 \cos t$$

(5)

The line l_2 passes through the origin and is perpendicular to l_1 .

The lines l_1 and l_2 intersect at the point Q.

(*b*) Show that, as t varies, an equation of the locus of Q is

$$(x^2 + y^2)^2 = 16x^2 - 4y^2$$

(8)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\pm\frac{a}{e}=8,\quad \pm ae=2$	B1, B1
	$\frac{a}{e}\times ae=a^2=16$	
	$a=4$	B1
	$b^2=a^2(1-e^2)=a^2-a^2e^2$	
	$\Rightarrow b^2=16-4=12$	M1
	$\Rightarrow b=\sqrt{12}=2\sqrt{3}$	A1 (5)
		5
2.	$x^2+4x+13=(x+2)^2+9$	B1
	$\int\frac{1}{(x+2)^2+9}dx=\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$	M1 A1
	$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}=\frac{1}{3}\left(\arctan 1-\arctan 0\right)$	M1
	$=\frac{\pi}{12}$	A1 (5)
		5
3(a)	$rhs=1+2\sinh^2 x=1+2\left(\frac{e^x-e^{-x}}{2}\right)^2$	M1
	$=\frac{2+e^{2x}-2+e^{-2x}}{2}$	M1
	$=\frac{e^{2x}+e^{-2x}}{2}=\cosh 2x=lhs$ *	A1 (3)
(b)	$1+2\sinh^2 x-3\sinh x=15$	
	$2\sinh^2 x-3\sinh x-14=0$	M1
	$\left(\sinh x+2\right)\left(2\sinh x-7\right)=0$	M1
	$\sinh x=-2,\frac{7}{2}$	A1
	$x=\ln\left(-2+\sqrt{(-2)^2+1)}\right)=\ln\left(-2+\sqrt{5}\right)$	M1
	$x=\ln\left(\frac{7}{2}+\sqrt{\left(\frac{7}{2}\right)^2+1)}\right)=\ln\left(\frac{7+\sqrt{53}}{2}\right)$	A1 (5)
		8

Question Number	Scheme	Marks
4(a)	$\int (a-x)^n \cos x\,dx = (a-x)^n \sin x + \int n(a-x)^{n-1} \sin x\,dx$	M1A1
	$\left[(a-x)^n \sin x\right]_0^a = 0$	A1
	$= -n(a-x)^{n-1}\cos x - \int n(n-1)(a-x)^{n-2}\cos x\,dx$	dM1
	$I_n = na^{n-1} - n(n-1)I_{n-2}$ *	A1 (5)
(b)	$I_2 = 2\left(\frac{\pi}{2}\right) - 2\int_0^{\frac{\pi}{2}} \cos x\,dx$	M1 A1
	$= \pi - 2[\sin x]_0^{\frac{\pi}{2}} = \pi - 2$	A1 (3)
		8
5(a)	$\frac{dy}{dx} = 2\text{ar}\cosh(3x) \times \frac{3}{\sqrt{9x^2-1}}$	M1A1A1
	$\sqrt{9x^2-1}\frac{dy}{dx} = 6\text{ar}\cosh(3x)$	
	$(9x^2-1)\left(\frac{dy}{dx}\right)^2 = 36(\text{ar}\cosh(3x))^2$	dM1
	$(9x^2-1)\left(\frac{dy}{dx}\right)^2 = 36y$ *	A1 (5)
(b)	$\left\{18x\left(\frac{dy}{dx}\right)^2 + (9x^2-1)\times 2\frac{dy}{dx}\times\frac{d^2y}{dx^2}\right\} = 36\frac{dy}{dx}$	M1 {A1} A1
	$(9x^2-1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18$ *	A1 (4)
		9

Question Number	Scheme	Marks
6(a)	$\begin{pmatrix}1 & 0 & 3\\ 0 & -2 & 1\\ k & 0 & 1\end{pmatrix}\begin{pmatrix}6\\1\\6\end{pmatrix}=\lambda\begin{pmatrix}6\\1\\6\end{pmatrix}$	
	$\begin{pmatrix}24\\4\\6k+6\end{pmatrix}=\begin{pmatrix}6\lambda\\\lambda\\6\lambda\end{pmatrix}$	
	Uses the first or second row to obtain $\lambda=4$	M1A1 (2)
(b)	Uses the third row and their $\lambda=4$ to obtain $6k+6=24\Rightarrow k=3$ *	M1 A1 (2)
(c)	$\begin{vmatrix}1-\lambda & 0 & 3\\ 0 & -2-\lambda & 1\\ 3 & 0 & 1-\lambda\end{vmatrix}=0$	
	$\Rightarrow(1-\lambda)\left((-2-\lambda)(1-\lambda)-0\right)-0\left(0(1-\lambda)-3\right)+3\left(0-3(-2-\lambda)\right)=0$	M1 A1
	$\Rightarrow(1-\lambda)(-2-\lambda)(1-\lambda)+9(2+\lambda)=(2+\lambda)\left(9-(1-\lambda)^2\right)=0$	
	$\left(\lambda^3-12\lambda-16=0\right)$	
	$\Rightarrow(\lambda+2)\left(\lambda^2-2\lambda-8\right)=0$	
	$\Rightarrow(\lambda+2)(\lambda+2)(\lambda-4)=0$	M1
	$\lambda=-2,4$	A1 (4)
(d)	Parametric form of l_1 : $(t+2,-3t,4t-1)$	M1
	$\begin{pmatrix}1 & 0 & 3\\ 0 & -2 & 1\\ 3 & 0 & 1\end{pmatrix}\begin{pmatrix}t+2\\-3t\\4t-1\end{pmatrix}=\begin{pmatrix}13t-1\\10t-1\\7t+5\end{pmatrix}$	M1 A1
	Cartesian equations of l_2 : $\dfrac{x+1}{13}=\dfrac{y+1}{10}=\dfrac{z-5}{7}$	ddM1A1(5)
		13

Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$	M1 A2(1,0)
	$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 5$	
	$\mathbf{r} \bullet \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1A1 (5)
(b)	Equation of l is $\mathbf{r} = \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$	M1
	At intersection $\begin{pmatrix} 6+t \\ 13+4t \\ 5+2t \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1
	$\Rightarrow 6+t+4(13+4t)+2(5+2t)=5 \Rightarrow t=-3$	M1
	N is $(3,1,-1)$ *	A1 (4)
(c)	$\overrightarrow{PN} \cdot \overrightarrow{PR} = (-3\mathbf{i}-12\mathbf{j}-6\mathbf{k}) \cdot (-5\mathbf{i}-13\mathbf{j}-3\mathbf{k}) = 189$	M1 A1ft
	$\sqrt{9+144+36}\sqrt{25+169+9}\cos NPR = 189$	A1
	$NX = NP\sin NPR = \sqrt{189}\sin NPR = 3.61$	M1A1 (5) **14**

Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt}=4\sec t\tan t \quad \frac{dy}{dt}=2\sec^2 t$	B1 (both)
	$\frac{dy}{dx}=\frac{2\sec^2 t}{4\sec t\tan t} \quad \left(=\frac{1}{2\sin t}\right)$	M1
	$y-2\tan t=\frac{1}{2\sin t}(x-4\sec t)$	M1 A1
	$2y\sin t-\frac{4\sin^2 t}{\cos t}=x-\frac{4}{\cos t}$	
	$2y\sin t=x-\frac{4-4\sin^2 t}{\cos t}=x-4\cos t \quad *$	A1 (5)
(b)	Gradient of l_2 is $-2\sin t$	M1
	$y=-2x\sin t \quad (2)$	A1
	$2(-2x\sin t)\sin t=x-4\cos t\Rightarrow x=\frac{4\cos t}{1+4\sin^2 t} \quad (1)$	M1 A1
	$y=\frac{-8\sin t\cos t}{1+4\sin^2 t}$	M1 A1
	$(x^2+y^2)^2=\left(\frac{16\cos^2 t}{(1+4\sin^2 t)^2}+\frac{64\sin^2 t\cos^2 t}{(1+4\sin^2 t)^2}\right)^2$	
	$=\frac{256\cos^4 t}{(1+4\sin^2 t)^4}(1+4\sin^2 t)^2=\frac{256\cos^4 t}{(1+4\sin^2 t)^2}$	M1
	$16x^2-4y^2=\frac{256\cos^2 t}{(1+4\sin^2 t)^2}-\frac{256\sin^2 t\cos^2 t}{(1+4\sin^2 t)^2}=\frac{256\cos^4 t}{(1+4\sin^2 t)^2}$	A1 (8) **13**